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BUCKLING OF CIRCULAR CYLINDRICAL SHELLS
WITH MULTIPLE ORTHOTROPIC LAYERS and ECCENTRIC STIFFENERS

by

ROBERT M. JONES

SEPTEMBER 1967

Prepared for
SPACE AND MISSILE SYSTEMS ORGANIZATION
AIR FORCE SYSTEMS COMMAND
Air Force Unit Post Office
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AEROSPACE CORPORATION
San Bernardino Operations

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FOREWORD

This report by Aerospace Corporation, San Bernardino Operations has been done under Contract No. F04695-67-C-0158 as TR-0158(53820-10)-1. The Air Force program monitor is Major W. D. Ohlemeier, USAF (SMYAC). The dates of research for this report include the period April 1967 through August 1967. This report was submitted by the author in August 1967.

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UNCLASSIFIED ABSTRACT

BUCKLING OF CIRCULAR CYLINDRICAL SHELLS WITH MULTIPLE ORTHOTROPIC LAYERS AND ECCENTRIC STIFFENERS, by Robert M. Jones

TR-0158(S3820-10)-1
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An exact solution is derived for the buckling of a circular cylindrical shell with multiple orthotropic layers and eccentric stiffeners under axial compression, lateral pressure, or any combination thereof. Classical stability theory (membrane prebuckled shape) is used for simply supported edge boundary conditions. The present theory enables the study of coupling between bending and extension due to the presence of different layers in the shell and to the presence of eccentric stiffeners. Previous approaches to stiffened multilayered shells are shown to be erratic in the prediction of buckling results due to neglect of coupling between bending and extension. (Unclassified Report)

CONTENTS

FIGURES

TABLES

NOMENCLATURE[1]

a	= ring spacing (Figure 1)
A	= cross-sectional area of a stiffener
A_{ij}	= coefficients in stability criterion [Eq. (18)]
b	= stringer spacing (Figure 1)
B_{ij}	= extensional stiffness of the layered shell
$B_x(B_y)$	= extensional stiffness of the orthotropic stiffness layer in the x-(y-) direction
B_{xy}	= in-plane shearing stiffness of the orthotropic stiffness layer
C_{ij}	= coupling stiffness of the layered shell
D_{ij}	= bending stiffness of the layered shell
$D_x(D_y)$	= bending stiffness of the orthotropic stiffness layer in the x-(y-) direction
D_{xy}	= twisting stiffness of the orthotropic stiffness layer
E	= Young's modulus of a stiffener
E_{xx}^k, E_{yy}^k	= Young's moduli in x and y directions, respectively, of the k^{th} shell layer
G	= shearing modulus, $E/\{2(1+\nu)\}$, of a stiffener
G_{xy}^k	= shearing modulus of the k^{th} shell layer in x-y plane
I	= moment of inertia of a stiffener about its centroid
J	= torsional constant of a stiffener

[1] A comma indicates partial differentiation with respect to the subscript following the comma. The prefix δ denotes the variation during buckling of the symbol which follows.

NOMENCLATURE (Continued)

K_{ij}^{k}	= function of material properties of the k^{th} layer [Eq. (2)]
L	= length of circular cylindrical shell (Figure 1)
m	= number of axial buckle halfwaves
M_x, M_y, M_{xy}, M_{yx}	= moments per unit length
n	= number of circumferential buckle waves
N	= number of layers
N_x, N_y, N_{xy}	= in-plane forces per unit length
$\overline{N}_x$, $\overline{N}_y$	= applied axial and circumferential forces per unit length
p	= external or hydrostatic pressure
R	= shell reference surface radius (Figures 1 and 2)
t_k	= thickness of k^{th} shell layer
u, v, w	= axial, circumferential, and radial displacements from a membrane prebuckled shape
x, y, z	= axial, circumferential, and radial coordinates on shell reference surface (Figure 1)
$\overline{z}$	= distance from stiffener centroid to shell reference surface (Figure 1), positive when stiffener on outside
ϵ_x, ϵ_y, γ_{xy}	= strains
ϵ_1, ϵ_2, ϵ_3	= variations in reference surface strains [Eq. (5)]
δ_k	distance from inner surface of layered shell to outer surface of k^{th} layer
Δ	distance from inner surface of layered shell to reference surface

NOMENCLATURE (Continued)

ν_{xy}^{k} (ν_{yx}^{k})	= Poisson's ratio for contraction in the y(x) direction due to tension in the x(y) direction
ν_{xyB} (ν_{yxB})	= so-called extensional Poisson's ratio for contraction in the y-(x-) direction due to tension in the x-(y-) direction
ν_{xyD} (ν_{yxD})	= so-called bending Poisson's ratio for curvature in the y-(x-) direction due to moment in the x-(y-) direction
σ_x, σ_y, τ_{xy}	= stresses
χ_1, χ_2, χ_3	= variations in reference surface curvatures [Eq. (6)]

Superscript

k	= k^{th} shell layer

Subscripts

k	= k^{th} shell layer
r	= ring
s	= stringer

SECTION I

INTRODUCTION

The first work in the area of stability of eccentrically stiffened shells was done by Van der Neut (Ref. 1) about twenty years ago. However, his conclusion that the buckling load under axial compression of an externally stiffened shell can be as high as two or three times that of an internally stiffened shell went essentially unnoticed. More recently, Baruch and Singer (Ref. 2) and Block, Card, and Mikulas (Ref. 3) presented theories which are considered basic in the field. Since 1965, work in the area of eccentrically stiffened shells has expanded so much that it is impractical to mention more than a few significant papers. McElman, Mikulas, and Stein (Ref. 4) extended the original work to include the effect of stiffeners on vibration and flutter. Correlation between theory and experiment was reported for static buckling loads by Card and Jones (Ref. 5). The effect of initial imperfections was considered by Hutchinson and Amazigo (Ref. 6). Block (Ref. 7) treated discrete ring spacing, prebuckling deformation, and load eccentricity. Finally, plastic buckling was discussed by Jones (Ref. 8).

The object of the present paper is to extend previous theories to consideration of stability of circular cylindrical shells with multiple orthotropic layers and eccentric stiffeners (see Figure 1). Classical stability theory, which implies a membrane prebuckled shape, is used for the simply supported edge boundary conditions $\delta N_x = v = w = \delta M_x = 0$. The layers have orthotropic material

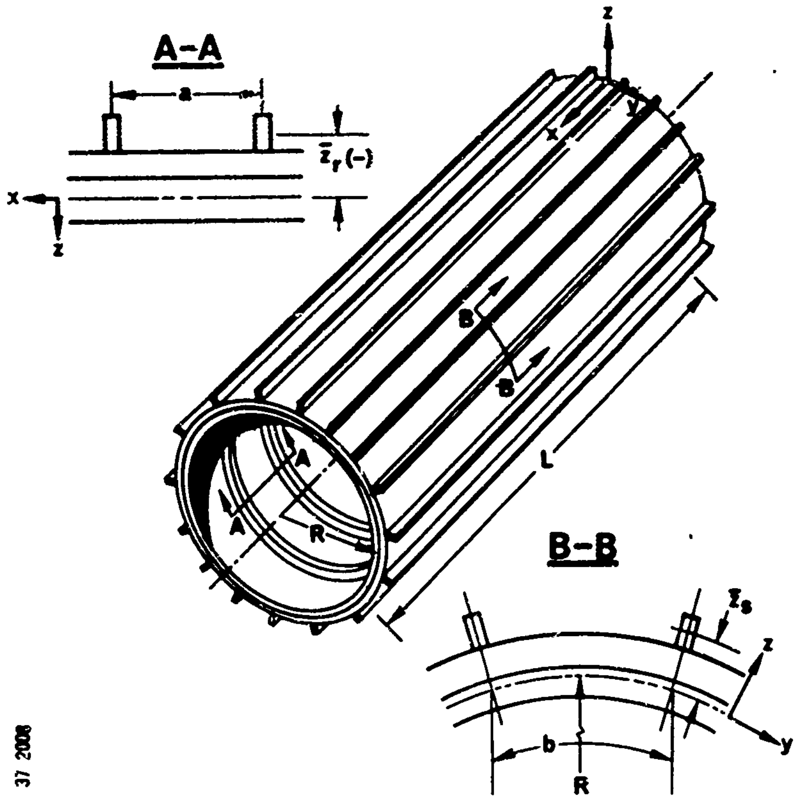

Figure 1. Stiffened Multilayered Shell

properties with the principal axes of orthotropy coincident with the shell coordinate directions. In accordance with most previous theories, the stiffeners are treated as isotropic one-dimensional beam elements and are averaged or "smeared out" over the stiffener spacing. The torsional rigidity of the stiffeners is accounted for in an approximate manner. The present theory enables the study of coupling between bending and extension due to the presence of different layers in the shell and to the presence of eccentric stiffeners.

SECTION II

DERIVATION OF THEORY

Expressions are obtained for the variations of stresses during buckling in the k^{th} layer of a multilayered shell in terms of the variations of strains during buckling. Subsequently, the variations of stresses are integrated over the shell and stiffeners in order to obtain expressions for the variations in forces and moments during buckling. Finally, the variations in forces and moments are substituted in Donnell-type stability differential equations which are then solved to yield a closed-form stability criterion in terms of the geometric and material properties of the stiffened multilayered circular cylindrical shell.

A. ORTHOTROPIC STRESS-STRAIN RELATIONS

The stress-strain relations for an orthotropic material can be written as

$$\left.\begin{aligned} \sigma_x^k &= K_{11}^k \epsilon_x + K_{12}^k \epsilon_y \\ \sigma_y^k &= K_{12}^k \epsilon_x + K_{22}^k \epsilon_y \\ \tau_{xy}^k &= K_{33}^k \gamma_{xy} \end{aligned}\right\} \qquad (1)$$

where

$$\left.\begin{aligned} K_{11}^k &= E_{xx}^k \Big/ \left(1 - \nu_{xy}^k \nu_{yx}^k\right) \\ K_{12}^k &= \nu_{xy}^k E_{xx}^k \Big/ \left(1 - \nu_{xy}^k \nu_{yx}^k\right) \\ K_{22}^k &= E_{yy}^k \Big/ \left(1 - \nu_{xy}^k \nu_{yx}^k\right) \\ K_{33}^k &= G_{xy}^k \end{aligned}\right\} \qquad (2)$$

wherein the superscript k denotes the k^{th} layer. The quantity E_{xx}^k (E_{yy}^k) is Young's modulus in the x (y) direction, G_{xy}^k is the shear modulus in the x-y plane, and ν_{xy}^k (ν_{yx}^k) is the Poisson's ratio for contraction in the y (x) direction due to tension in the x (y) direction. There are apparently five material constants per layer; however, because of the reciprocal relations ($\nu_{xy}^k E_{xx}^k = \nu_{yx}^k E_{yy}^k$), there are actually only four independent constants.

B. VARIATIONS OF STRESSES AND STRAINS DURING BUCKLING

During buckling, the stresses vary from their prebuckling values. Let the variation be denoted by δ ; then, from Eq. (1)

$$\left.\begin{aligned} \delta\sigma_x^k &= K_{11}^k \delta\epsilon_x + K_{12}^k \delta\epsilon_y \\ \delta\sigma_y^k &= K_{12}^k \delta\epsilon_x + K_{22}^k \delta\epsilon_y \\ \delta\tau_{xy}^k &= K_{33}^k \delta\gamma_{xy} \end{aligned}\right\} \tag{3}$$

where $\delta\epsilon_x$, $\delta\epsilon_y$, and $\delta\gamma_{xy}$ denote the corresponding variations in the strains during buckling. Because of the Kirchhoff-Love hypothesis, the variations in strains during buckling are

$$\left.\begin{aligned} \delta\epsilon_x &= \epsilon_1 + z\chi_1 \\ \delta\epsilon_y &= \epsilon_2 + z\chi_2 \\ \delta\gamma_{xy} &= \epsilon_3 + z\chi_3 \end{aligned}\right\} \tag{4}$$

The z coordinate is measured from an arbitrary reference surface (see Figure 1). In Eq. (4), ϵ_1, ϵ_2, and ϵ_3 are the variations of the reference surface strains

$$\left.\begin{aligned} \epsilon_1 &= u,_x \\ \epsilon_2 &= v,_y + w/R \\ \epsilon_3 &= u,_y + v,_x \end{aligned}\right\} \tag{5}$$

and χ_1, χ_2, and χ_3 are the variations of the reference surface curvatures

$$\left.\begin{aligned} \chi_1 &= -w,_{xx} \\ \chi_2 &= -w,_{yy} \\ \chi_3 &= -2w,_{xy} \end{aligned}\right\} \qquad (6)$$

Upon substitution of Eq. (4), the variations in stresses in the k^{th} layer can be written as

$$\left.\begin{aligned} \delta\sigma_x^{\,k} &= K_{11}^k (\epsilon_1 + z\chi_1) + K_{12}^k (\epsilon_2 + z\chi_2) \\ \delta\sigma_y^{\,k} &= K_{12}^k (\epsilon_1 + z\chi_1) + K_{22}^k (\epsilon_2 + z\chi_2) \\ \delta\tau_{xy}^{\,k} &= K_{33}^k (\epsilon_3 + z\chi_3) \end{aligned}\right\} \qquad (7)$$

C. VARIATIONS OF FORCES AND MOMENTS DURING BUCKLING

The variations of forces and moments during buckling are obtained by integration of the variations of stresses over the shell layers and stiffeners. The effect of the stiffeners on the variations of forces and moments is averaged or "smeared out" over the stiffener spacing.

$$\left.\begin{aligned} \delta N_x &= \sum_{k=1}^{N} \int_{t_k} \delta\sigma_x^{\,k}\, dz + \frac{1}{b}\int_{A_s} \delta\sigma_x\, dA_s \\ \delta N_y &= \sum_{k=1}^{N} \int_{t_k} \delta\sigma_y^{\,k}\, dz + \frac{1}{a}\int_{A_r} \delta\sigma_y\, dA_r \\ \delta N_{xy} &= \sum_{k=1}^{N} \int_{t_k} \delta\tau_{xy}^{\,k}\, dz \end{aligned}\right\} \qquad (8)$$

$$
\left.
\begin{aligned}
\delta M_x &= \sum_{k=1}^{N} \int_{t_k} \delta\sigma_x^{\,k}\, z dz + \frac{1}{b}\int_{A_s} \delta\sigma_x\, z dA_s \\
\delta M_y &= \sum_{k=1}^{N} \int_{t_k} \delta\sigma_y^{\,k}\, z dz + \frac{1}{a}\int_{A_r} \delta\sigma_y\, z dA_r \\
\delta M_{xy} &= -\sum_{k=1}^{N} \int_{t_k} \delta\tau_{xy}^{\,k}\, z dz - \frac{G_s J_s}{2b} \chi_3 \\
\delta M_{yx} &= \sum_{k=1}^{N} \int_{t_k} \delta\tau_{xy}^{\,k}\, z dz + \frac{G_r J_r}{2a} \chi_3
\end{aligned}
\right\} \qquad (9)
$$

where t_k denotes the thickness of the k^{th} layer and N is the number of layers. The variations of stresses for the stiffeners are based on uniaxial isotropic reductions of the orthotropic stress-strain relations. The integrations in Eqs. (8) and (9) yield

$$
\left.
\begin{aligned}
\delta N_x &= (B_{11} + E_s A_s/b)\,\epsilon_1 + B_{12}\epsilon_2 + (C_{11} + \bar{z}_s E_s A_s/b)\,\chi_1 \\
&\quad + C_{12}\chi_2 \\
\delta N_y &= B_{12}\epsilon_1 + (B_{22} + E_r A_r/a)\,\epsilon_2 + C_{12}\chi_1 \\
&\quad + (C_{22} + \bar{z}_r E_r A_r/a)\,\chi_2 \\
\delta N_{xy} &= B_{33}\epsilon_3 + C_{33}\chi_3
\end{aligned}
\right\} \qquad (10)
$$

$$
\left.
\begin{aligned}
\delta M_x &= (C_{11} + \bar{z}_s E_s A_s/b)\,\epsilon_1 + C_{12}\epsilon_2 + (D_{11} + \bar{z}_s^2 E_s A_s/b \\
&\quad + E_s I_s/b)\,\chi_1 + D_{12}\chi_2 \\
\delta M_y &= C_{12}\epsilon_1 + (C_{22} + \bar{z}_r E_r A_r/a)\,\epsilon_2 + D_{12}\chi_1 \\
&\quad + (D_{22} + \bar{z}_r^2 E_r A_r/a + E_r I_r/a)\,\chi_2
\end{aligned}
\right\} \qquad (11)
$$

Cont. next page

$$\left.\begin{aligned} \delta M_{xy} &= -C_{33}\epsilon_3 - (D_{33} + G_s J_s/2b)X_3 \\ \delta M_{yx} &= C_{33}\epsilon_3 + (D_{33} + G_r J_r/2a)X_3 \end{aligned}\right\}$$ Cont. from prev. page

where

$$\left.\begin{aligned} B_{ij} &= \sum_{k=1}^{N} K_{ij}^{k}(\delta_k - \delta_{k-1}) \\ C_{ij} &= \frac{1}{2}\sum_{k=1}^{N} K_{ij}^{k}\left[\left(\delta_k^2 - \delta_{k-1}^2\right) - 2\Delta(\delta_k - \delta_{k-1})\right] \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{N} K_{ij}^{k}\left[\left(\delta_k^3 - \delta_{k-1}^3\right) - 3\Delta\left(\delta_k^2 - \delta_{k-1}^2\right)\right. \\ &\quad \left. + 3\Delta^2(\delta_{k} - \delta_{k-1})\right] \end{aligned}\right\} \quad (12)$$

The stiffnesses in Eq. (12) are due to Ambartsumyan (Ref. 9) and depend on the location of the reference surface (see Figure 2). The reference surface can be changed by varying Δ in order to study different loading and boundary conditions. Geier (Ref. 10) obtains expressions which are more simple in appearance than Eq. (10), but which are more difficult to utilize.

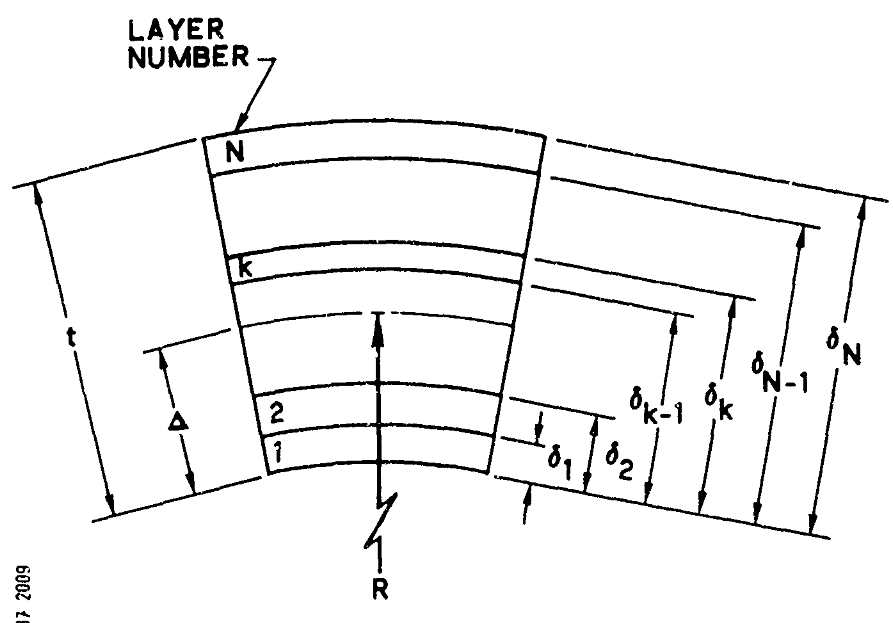

Figure 2. Cross Section of an N-Layered Shell

D. STABILITY DIFFERENTIAL EQUATIONS

The Donnell-type stability differential equations for circular cylindrical shells subjected to combinations of axial compression and lateral pressure are

$$\left.\begin{aligned} &\delta N_{x,x} + \delta N_{xy,y} = 0 \\ &\delta N_{xy,x} + \delta N_{y,y} = 0 \\ &- \delta M_{x,xx} + \delta M_{xy,xy} - \delta M_{yx,xy} - \delta M_{y,yy} + \delta N_y/R \\ &+ \overline{N}_x w,_{xx} + \overline{N}_y w,_{yy} = 0 \end{aligned}\right\} \qquad (13)$$

and the alternative force and geometric boundary conditions at $x = 0$ and L are chosen from the following sixteen possibilities (any set of four alternatives in the following pairs constitutes a set of boundary conditions).

$$\left.\begin{aligned} \delta N_x &= 0 \quad \text{or} \quad u = 0 \\ \delta N_{xy} &= 0 \quad \text{or} \quad v = 0 \\ \delta M_{x,x} + \delta M_{yx,y} + \overline{N}_x w,_x &= 0 \quad \text{or} \quad w = 0 \\ \delta M_x &= 0 \quad \text{or} \quad w,_x = 0 \end{aligned}\right\} \qquad (14)$$

Upon substitution of the expressions for the variations of forces and moments during buckling [Eqs. (10) and (11)] and the variations of reference surface strains and curvatures [Eqs. (5) and (6)] , the stability differential equations become

$$\left.\begin{aligned}
&(B_{11} + E_s A_s/b)\, u,_{xx} + B_{12}\,(v,_{xy} + w,_x/R) \\
&\quad + B_{33}\,(u,_{yy} + v,_{xy}) - (C_{11} + \bar{z}_s E_s A_s/b)\, w,_{xxx} \\
&\quad - (C_{12} + 2C_{33})\, w,_{xyy} = 0 \\
&B_{12} u,_{xy} + (B_{22} + E_r A_r/a)\,(v,_{yy} + w,_y/R) \\
&\quad + B_{33}\,(u,_{xy} + v,_{xx}) - (C_{12} + 2C_{33})\, w,_{xxy} \\
&\quad - (C_{22} + \bar{z}_r E_r A_r/a)\, w,_{yyy} = 0 \\
&(B_{12}/R)\, u,_x - (C_{11} + \bar{z}_s E_s A_s/b)\, u,_{xxx} - (C_{12} + 2C_{33})(u,_{xyy} \\
&+ v,_{xxy}) + (1/R)(B_{22} + E_r A_r/a)(v,_y + w/R) \\
&+ (C_{22} + \bar{z}_r E_r A_r/a)\, v,_{yyy} - (2C_{12}/R)\, w,_{xx} \\
&+ (2/R)(C_{22} + \bar{z}_r E_r A_r/a)\, w,_{yy} + (D_{11} + \bar{z}_s^2 E_s A_s/b \\
&+ E_s I_s/b)\, w,_{xxxx} + (4D_{33} + 2D_{12} + G_s J_s/b \\
&+ G_r J_r/a)\, w,_{xxyy} + (D_{22} + \bar{z}_r^2 E_r A_r/a + E_r I_r/a)\, w,_{yyyy} \\
&+ \bar{N}_x w,_{xx} + \bar{N}_y\, w,_{yy} = 0
\end{aligned}\right\} \qquad (15)$$

E. STABILITY CRITERION

It is desired to find the solution to the stability differential equations for the simply supported edge boundary conditions

$$\delta N_x = v = w = \delta M_x = 0 \qquad (16)$$

The following buckling displacements satisfy the boundary conditions of Eq. (16):

$$\left.\begin{aligned} u &= \bar{u}\cos(m\pi x/L)\cos(ny/R) \\ v &= \bar{v}\sin(m\pi x/L)\sin(ny/R) \\ w &= \bar{w}\sin(m\pi x/L)\cos(ny/R) \end{aligned}\right\} \tag{17}$$

(where $\bar{u}$, $\bar{v}$, and $\bar{w}$ are the amplitudes of the buckling displacements) and are substituted in the stability differential equations [Eq. (15)] . In order to obtain a nontrivial solution to the resulting equations, the determinant of the coefficients of $\bar{u}$, $\bar{v}$, and $\bar{w}$ must be zero, and the following stability criterion results:

$$\left.\begin{aligned} \bar{N}_x(m\pi/L)^2 + \bar{N}_y(n/R)^2 &= A_{33} + A_{23}\left(\frac{A_{13}A_{12} - A_{11}A_{23}}{A_{11}A_{22} - A_{12}^2}\right) \\ &+ A_{13}\left(\frac{A_{12}A_{23} - A_{13}A_{22}}{A_{11}A_{22} - A_{12}^2}\right) \end{aligned}\right\} \tag{18}$$

where

$$\left.\begin{aligned} A_{11} &= (B_{11} + E_sA_s/b)(m\pi/L)^2 + B_{33}(n/R)^2 \\ A_{12} &= (B_{12} + B_{33})(m\pi/L)(n/R) \\ A_{13} &= (B_{12}/R)(m\pi/L) + (C_{11} + \bar{z}_sE_sA_s/b)(m\pi/L)^3 \\ &+ (C_{12} + 2C_{33})(m\pi/L)(n/R)^2 \\ A_{22} &= B_{33}(m\pi/L)^2 + (B_{22} + E_rA_r/a)(n/R)^2 \\ A_{23} &= (C_{12} + 2C_{33})(m\pi/L)^2(n/R) + (1/R)(B_{22} + E_rA_r/a)(n/R) \\ &+ (C_{22} + \bar{z}_rE_rA_r/a)(n/R)^3 \end{aligned}\right\} \quad (19)$$

Cont. next page

$$
\left.\begin{aligned}
A_{33} = {} & (D_{11} + E_s I_s/b + \bar{z}_s{}^2 E_s A_s/b)(m\pi/L)^4 \\
& + (4D_{33} + 2D_{12} + G_s J_s/b + G_r J_r/a)(m\pi/L)^2 (n/R)^2 \\
& + (D_{22} + E_r I_r/a + \bar{z}_r{}^2 E_r A_r/a)(n/R)^4 + (2C_{12}/R)(m\pi/L)^2 \\
& + (2/R)(C_{22} + \bar{z}_r E_r A_r/a)(n/R)^2 + (1/R^2)(B_{22} + E_r A_r/a)
\end{aligned}\right\} \quad \text{(19) Cont. from prev. page}
$$

The solution represented by Eq. (18) reduces to the solution of Ref. 3 for stiffened single-layered isotropic circular cylindrical shells. In addition, stiffener eccentricity is more obviously accounted for in the foregoing derivation than in the work of Geier (Ref. 10)

The buckling load under axial compression is obtained from Eq. (18) by equating $\bar{N}_y$ to zero and solving for $\bar{N}_x$. Similarly, the buckling load under lateral pressure is obtained by equating $\bar{N}_x$ to zero and solving for $\bar{N}_y(\bar{N}_y = pR/t)$. Finally, the buckling load under hydrostatic pressure is obtained by equating $\bar{N}_x$ to $\bar{N}_y/2$ and solving for $\bar{N}_y$. In addition, if $\bar{N}_x(\bar{N}_y)$ is fixed, the critical value of $\bar{N}_y(\bar{N}_x)$ can be found. In this manner, an interaction curve between axial compression and lateral pressure can be obtained.

Because of the numerous parameters in Eq. (18) and the need to investigate a large range of buckling modes to determine the lowest buckling load, it is necessary from a practical standpoint to use a digital computer for numerical work. In the computer program (see Appendixes A and C), for a given number of axial halfwaves, m, and circumferential waves, n, in the buckled shape, the appropriate buckling load is found. The number n is varied in an inner DO loop for a fixed m until all relative minima of the buckling load are found within a given range of values of n. The number m is then varied in an cuter DO loop so that all relative minima are found. Finally, the absolute minimum buckling load is selected from the relative minima.

SECTION III

NUMERICAL EXAMPLE

Because of the many geometrical properties in the theory, meaningful general results cannot be presented. Accordingly, a specific numerical example is given to illustrate application of the theory. The results are compared with results of previous approaches to the same problem.

For this example, the stability of a ring-stiffened circular cylindrical shell with two isotropic layers under hydrostatic pressure is considered. The properties of the layers are

$E_1 = 44 \times 10^6$ psi $E_2 = 2 \times 10^6$ psi

$\nu_1 = 0$ $\nu_2 = 0.4$

$t_1 = 0.04$ in. $t_2 = 0.3$ in.

The rings are of rectangular cross section with a height of 0.25 inch and a thickness of 0.06 inch. The rings are on the inner surface of layer one and have the same material properties as layer one. The shell has a length of 12 inches and a radius of 6 inches to the middle surface of layer one (which, in this case, is also the reference surface).

The hydrostatic buckling pressure of the above configuration is shown as the solid line in Figure 3 as a function of ring spacing. The results shown are for general instability (buckling in which the rings participate). The buckling pressures for panel instability (buckling between rings) are much higher than the present results and, hence, do not govern the stability of the present configuration. Other failure criteria, e.g., yielding, are ignored for the purposes of this illustration of the present analysis technique. The dashed curve in Figure 3 represents

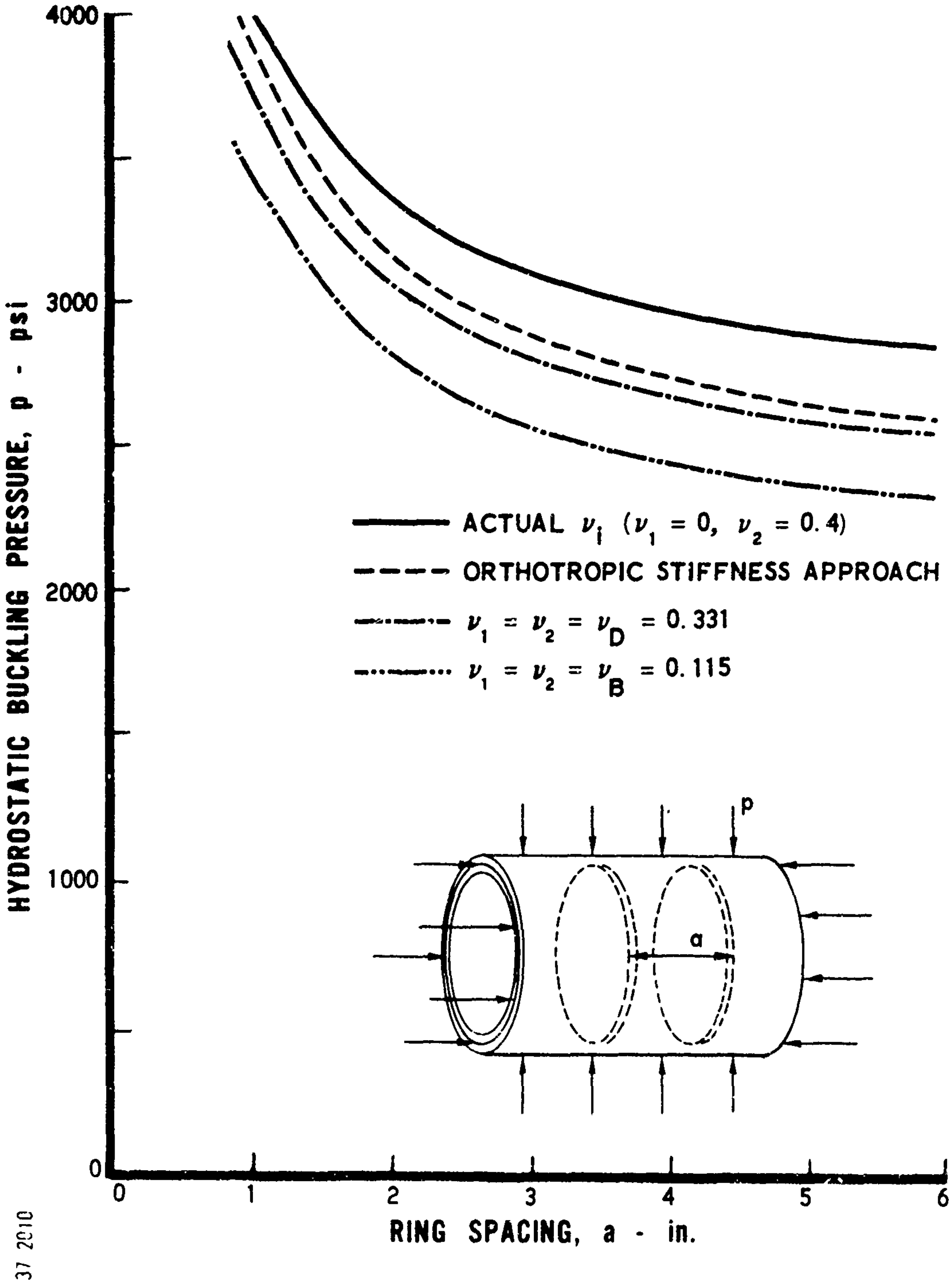

Figure 3. Hydrostatic Buckling Pressure of a Ring-Stiffened, Two-Layered Circular Cylindrical Shell

an orthotropic stiffness approach to the problem and is from 3 to 9 percent lower than the results from the present theory. These lower results are due to neglect of coupling between bending and extension of the layered shell and the eccentric stiffeners in the orthotropic stiffness approach. The solid curve with a single dot represents a stiffened shell with a single equivalent Poisson's ratio for bending ($\nu_D = 0.331$) used in both layers (Ref. 11) and is from 7 to 11 percent lower than the results of the present theory. Finally, the solid curve with two dots represents a stiffened shell with a single equivalent Poisson's ratio for extension ($\nu_B = 0.115$) used in both layers (Ref. 11) and is from 14 to 18 percent lower than the results of the present theory. The lower results for ν_D and ν_B are due to neglect of coupling between bending and extension of the two shell layers. Note that all approaches previous to the present theory are conservative for this example, i.e., they yield lower buckling pressures than can actually be realized by the stiffened shell. For other problems, the previous approaches can yield unconservative results (Ref. 11). Thus, the importance of coupling between bending and extension should not be overlooked.

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SECTION IV

CONCLUDING REMARKS

An exact solution, within the framework of classical stability theory, is derived for the buckling of a circular cylindrical shell with multiple orthotropic layers and eccentric stiffeners under axial compression, lateral pressure, or any combination thereof. The simply supported edge boundary conditions are $\delta N_x = v = w = \delta M_x = 0$. Thus, the present solution can be regarded as a lower bound on results for practical shells if initial imperfections, prebuckling deformations, and effects of discrete stiffener spacing are ignored.

A numerical example is given to illustrate the effect of coupling between bending and extension due to the presence of different layers in the shell and to the presence of eccentric stiffeners. Comparison of the present theory is made with previous approaches such as use of a single equivalent Poisson's ratio in all layers of a layered shell and orthotropic treatment of stiffened shells. The buckling predictions of the previous approaches, in which coupling is neglected, are seen to be erratic in that they are sometimes conservative and sometimes unconservative. Thus, the importance of coupling between bending and extension should not be overlooked.

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APPENDIX A

DESCRIPTION OF COMPUTER PROGRAM

A computer program was written to evaluate the closed-form stability criterion, Eq. (18), for an arbitrary range of values of the buckling mode parameters m and n and to select subsequently the lowest buckling load in the range. Program card decks are available upon request to the Aerospace Corporation, San Bernardino Operations, Mathematics and Computation Center. Specific characteristics and the usage of the program are described in the following discussion.

A.1 GENERAL CHARACTERISTICS

The basic capability of the program is represented by Eq. (18) which is valid for the stability of circular cylindrical shells with multiple orthotropic layers and eccentric stiffeners under axial compression, lateral pressure, or hydrostatic pressure. The boundary conditions at the edges are $\delta N_x = v = w = \delta M_x = 0$. The orthotropic material properties for each layer of thickness, t^k, are E^k_{xx}, E^k_{yy}, ν^k_{xy}, ν^k_{yx} (recall that because of the reciprocal relations only three are independent) and G^k_{xy}. It should be noted that the principal axes of orthotropy must coincide with the shell coordinates. The geometrical properties for the stiffeners are: area (A), moment of inertia about the stiffener centroid (I), eccentricity ($\bar{z}$), torsional constant (J), and spacing. The stiffeners are isotropic; hence, E and ν are the only material properties required.

Because mainly algebraic operations are performed in the program, the execution time is very small (less than 1 second per case).

As far as is possible, mnemonic representations are used throughout the program.

A.2 ORTHOTROPIC STIFFNESS LAYER, ØSL

Block, Card, and Mikulas included an orthotropic stiffness layer in their theory (Ref. 3) in order to treat corrugated shells, etc. In the present program, a similar layer can be used in place of the first layer of the multilayered shell if the reference surface is chosen to be the middle surface of the orthotropic stiffness layer. The orthotropic stiffness definitions reduce to the usual definitions for an isotropic shell, i.e.,

$$\left.\begin{aligned} B_x &= B_y = B = Et/(1-\nu^2) \\ B_{xy} &= [(1-\nu)/2]\ B = Et/[2(1+\nu)] \\ D_x &= D_y = D = Et^3/[12(1-\nu^2)] \\ D_{xy} &= [(1-\nu)/2]\ D = Et^3/[24(1+\nu)] \\ \nu_{xyB} &= \nu_{yxB} = \nu_{xyD} = \nu_{yxD} = \nu \end{aligned}\right\} \qquad \text{(A-1)}$$

The orthotropic stiffnesses must satisfy the reciprocal relations $\nu_{xyB}B_x = \nu_{yxB}B_y$ and $\nu_{xyD}D_x = \nu_{yxD}D_y$. It is important to note that ν_{xyB}, etc are, in some cases, not solely material properties, but are also affected by the geometry, e.g., corrugated or layered shells.

The orthotropic stiffness layer was used to describe the two-layered eccentrically stiffened shell in Section III, Numerical Example, in order to obtain the curve labeled Orthotropic Stiffness Approach in Figure 3. Note that this approach neglects coupling between bending and extension of the stiffeners and the layered shell and also neglects coupling between bending and extension of the layers.

Eccentric stiffeners can be added to the orthotropic stiffness layer if the eccentricity is properly accounted for. The eccentricity, ZR or ZS, is ordinarily input as the distance from the centroid to the base of the stiffener. Subsequently, the eccentricity is adjusted in the program to be the distance from the centroid of the stiffener to the arbitrary reference surface of the layered shell. However, when the orthotropic stiffness layer (ØSL) is used, the reference surface is fixed at the middle surface of the ØSL. In order that the stiffener bend about the middle surface of the layer to which it is attached, it is necessary to modify the input eccentricity such that, when one-half the ØSL thickness is added, the eccentricity totals one-half the thickness of the layer to which it is attached plus the distance from the base to the centroid of the stiffener.

A.3 INPUT PARAMETERS

The following is a list of input parameters and their format and definitions:

```
C
C    *CARD 1     FORMAT(80H) - PROBLEM TITLE
C
C    *CARD 2     FORMAT(I10,6F10.0)
C       NL - NUMBER OF LAYERS INCLUDING ORTHOTROPIC STIFFNESS LAYER
C        *RESTRICTED TO 9 IN DIMENSION LN(9) AND BY FORMAT NO.8.  THE USUAL THIN
C           SHELL LIMITATIONS MUST BE TAKEN INTO CONSIDERATION AS WELL.
C       OSL -ORTHOTROPIC STIFFNESS LAYER
C         IF EQUAL TO 0.,NO OSL
C         IF EQUAL TO 1.,OSL REPLACES LAYER ONE
C       LOAD - CODE NAME FOR TYPE OF LOAD
C         IF EQUAL TO 1., AXIAL COMPRESSION
C         IF EQUAL TO 2., LATERAL PRESSURE
C         IF EQUAL TO 3., HYDROSTATIC PRESSURE
C       MO,MF - INITIAL AND FINAL VALUES OF M, THE NUMBER OF AXIAL HALF-WAVES
C        *MO CANNOT BE ZERO IN THE AXIAL AND HYDROSTATIC LOADING CONDITIONS.
C        *MO SHOULD BE 1 FOR FINITE LENGTH SHELLS.
C        *IF NO ABSOLUTE MINIMUM LOAD IS FOUND OR IF THE RELATIVE MINIMA ARE
C           DECREASING WHEN M=MF, A MESSAGE IS PRINTED STATING THAT THE RANGE
C           ON M IS INSUFFICIENT TO DETERMINE AN ABSOLUTE MINIMUM.
C        *THE INTERVAL (MO,4*MF) IS EXAMINED INDEPENDENTLY FOR THE AXISYMMETRIC
C           BUCKLING LOAD WHICH IS THEN PRINTED AND ALSO SAVED FOR COMPARISON
C           WITH THE ASYMMETRIC BUCKLING LOAD.
C        *THE LONGER THE SHELL, THE HIGHER MF MUST BE.
C       NO,NF - INITIAL AND FINAL VALUES OF N, THE NUMBER OF CIRCUMFERENTIAL
C               WAVES
C               *THE ENTIRE INTERVAL (NO,NF) IS EXAMINED EVEN IF A RELATIVE
C                  MINIMUM IS FOUND WITHIN THE INTERVAL.
C               *NO IS NORMALLY 2 BECAUSE A SEARCH FOR THE AXISYMMETRIC
C                  BUCKLING LOAD IS AUTOMATICALLY PROVIDED IN THE AXIAL
C                  AND HYDROSTATIC PRESSURE LOADING CONDITIONS.
C               *NO CANNOT BE ZERO IN THE LATERAL PRESSURE LOADING CONDITION.
C               *MO AND NO CANNOT BOTH BE ZERO IN THE HYDROSTATIC PRESSURE
C                  LOADING CONDITION.

C               *IF NO RELATIVE MINIMUM IS FOUND OR THE LOAD IS AGAIN
C                  DECREASING AFTER ONE MINIMUM HAS BEEN FOUND WHEN N=NF,
C                  A MESSAGE IS PRINTED STATING THAT THE INTEVAL IS INADEQUATE.
C               *THE THINNER THE SHELL, THE HIGHER NF MUST BE.
C
C    *CARDS 3 THROUGH NL+2 - FORMAT(7E10.3) - ORTHOTROPIC LAYER PROPERTIES
C       LN(I)    - LAYER NUMBER
C       EXX(I)   - MODULUS OF ELASTICITY OF THE ITH LAYER IN THE X-DIRECTION
C       EYY(I)   - MODULUS OF ELASTICITY OF THE ITH LAYER IN THE Y-DIRECTION
C       NUXY(I)  - POISSON'S RATIO FOR CONTRACTION IN THE Y-DIRECTION DUE TO
C                  TENSION IN THE X-DIRECTION
C       NUYX(I)  - POISSON'S RATIO FOR CONTRACTION IN THE X-DIRECTION DUE TO
C                  TENSION IN THE Y-DIRECTION
C         *NOTE THAT BY THE RECIPROCAL RELATIONS  NUXY*EXX=NUYX*EYY.
C       GXY(I)   - SHEAR MODULUS OF ITH LAYER FOR THE XY-PLANE.
C       T(I)     - THICKNESS OF THE ITH LAYER
C         *IF AN ORTHOTROPIC STIFFNESS LAYER IS USED, ALL PROPERTIES OF THE
C          FIRST LAYER ARE ZERO.
```

```
C     *CARD OSL*(NL+3) - FORMAT(5E10.3) - ORTHOTROPIC STIFFNESS LAYER PROPERTIES
C        BX    - EXTENSIONAL STIFFNESS IN X-DIRECTION
C        BY    - EXTENSIONAL STIFFNESS IN Y-DIRECTION
C        BXY   - SHEAR STIFFNESS IN XY-PLANE
C        NUXYB- EXTENSIONAL POISSON'S RATIO FOR CONTRACTION IN THE Y-DIRECTION
C               DUE TO TENSION IN THE X-DIRECTION.
C        TOSL  - MAXIMUM THICKNESS OF OSL (USED AS T(1) IN STIFFNESS EQUATIONS
C               FOR LAYERED CYLINDER)
C
C     *CARD OSL*(NL+4) - FORMAT(4E10.3) - OSL PROPERTIES, CONTINUED
C        DX    - BENDING STIFFNESS IN X-DIRECTION
C        DY    - BENDING STIFFNESS IN Y-DIRECTION
C        DXY   - TWISTING STIFFNESS OF XY-PLANE
C        NUXYD- BENDING POISSON'S RATIO FOR CURVATURE IN THE Y-DIRECTION
C               DUE TO MOMENT IN THE X-DIRECTION
C
C     *CARD NL+2*OSL+3 - FORMAT(6E10.3) - RING PROPERTIES
C        ER   - MODULUS OF ELASTICITY
C        AR   - CROSS-SECTIONAL AREA
C        ZR   - ECCENTRICITY (MEASURED NEGATIVELY INWARD FROM INNER SURFACE OF
C               COMPOSITE SHELL TO RING CENTROID IF RINGS ARE INTERNAL -
C               POSITIVELY OUTWARD FROM OUTER SURFACE IF RINGS ARE EXTERNAL)
C        IR   - MOMENT OF INERTIA OF RING ABOUT ITS OWN CENTROID
C        GRJR- SHEAR MODULUS*TORSION CONSTANT OF CROSS SECTION
C        A    - SPACING OF RINGS
C
C     *CARD NL+2*OSL+4 - FORMAT(6E10.3) - STRINGER PROPERTIES
C        ES,AS,ZS,IS,GSJS,B - CORRESPOND TO ABOVE RING PROPERTIES
C
C     *CARD NL+2*OSL+5 - FORMAT(3E10.3) - BASIC GEOMETRY
C        L     - LENGTH OF CIRCULAR CYLINDRICAL SHELL
C        R     - RADIUS TO REFERENCE SURFACE
C                 *MUST BE TO MIDDLE SURFACE OF OSL IF AN OSL IS PRESENT
C        DELTA- DISTANCE FROM INNER SURFACE OF LAYERED CYLINDER TO REFERENCE
C               SURFACE
C                 *MUST BE 1/2*OSL THICKNESS IF AN OSL PRESENT.
C                 *SHOULD GET DIFFERENT AXIAL BUCKLING LOADS WHEN DELTA VARIED.
C
```

A. 4 OUTPUT

The output for each case is printed on one page if the sum of the number of layers, LN, and the number of axial buckle halfwaves, M , does not exceed 25 and, if, in addition, there is no more than one relative minimum buckling load per value of M . If these conditions are not met, additional pages are used as needed.

First, a user-specified case identification is printed. Next, the input quantities are printed so that input errors can be identified. The orthotropic layer properties are printed and are followed by the orthotropic stiffness layer (ØSL) properties, if any. Next, the ring and stringer properties are printed. Finally, the basic geometry quantities, shell length, radius, and reference surface location, are printed.

After execution of the program, the buckling load for axisymmetric deformation (absolute minimum in the range from $M = 1$ to $M = 4*MF$) is printed along with the value of M at which it occurs. Subsequently, the asymmetric buckling loads (relative minima for each value of M for the range from $N = 2$ to $N = NF$) are printed. The final result is the absolute minimum (axisymmetric or asymmetric) buckling load for the entire range of M and N.

A typical output page is shown in Appendix B.

APPENDIX B

EXAMPLE PROBLEM

The example chosen here is the configuration discussed in Section III, Numerical Example, in the main body of the report, i.e., a ring-stiffened circular cylindrical shell with two isotropic layers under hydrostatic pressure. Pertinent geometrical and material properties are given in Section III. Ring spacing for this example is 3 inches. The input data are shown in Table B-I. Figure B-1 illustrates the input form, and the computer output is shown in Figure B-2.

Table B-I

INPUT DATA FOR EXAMPLE PROBLEM

CASE IDENTIFICATION: CONFIGURATION OF FIGURE 3 - ACTUAL NUI			
Symbol	Value	Symbol	Value
NL	2	LN (2)	2
ØSL	0	EXX(2)	2×10^6
LØAD	3	EYY(2)	2×10^6
MØ	1	NUXY(2)	0.4
MF	10	NUYX(2)	0.4
NØ	2	GXY(2)	0.7179×10^6
NF	20	T(2)	0.3
LN(1)	1	ER	44×10^6
EXX(1)	44×10^6	AR	0.015
EYY(1)	44×10^6	ZR	-0.125
NUXY(1)	0	IR	0.7812×10^{-4}
NUYX(1)	0	GRJR	396
GXY(1)	22×10^6	A	3
T(1)	0.04	L	12
		R	6
		DELTA	0.02

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80 COLUMN KEYPUNCH FORM

PROGRAMMER ________ KEYPUNCHED ________ VERIFIED ________ DATE ________ PAGE ____ OF ____

1–10	11–20	21–30	31–40	41–50	51–60	61–72	73–80
1 CØNFIGUR	ATION ØF F	IGURE 3 - A	CTUAL NUI				
2	0	3	1	10	2	20	
1.	44.+6	44.+6	0.	0.	22.+6	.04	
2.	2.+6	2.+6	.4	.4	.7179+6	.30	
44.+6	.015	-.125	.7812-4	396.	3.		
12.	6.	.02					

AEROSPACE FORM 2049

Figure B-1. Example Input Form

```
CONFIGURATION OF FIGURE 3 - ACTUAL NUI
ELASTIC BUCKLING OF SIMPLY SUPPORTED, ECCENTRICALLY STIFFENED CIRCULAR CYLINDRICAL SHELLS
WITH MULTIPLE ORTHOTROPIC LAYERS UNDER HYDROSTATIC PRESSURE

NO=  1.     MF= 10.     NO=  2.     NF= 20.

PROPERTIES OF 2 ORTHOTROPIC LAYERS
LAYER         EXX            EYY            NUXY            NUYX            GXY            T
 1.      0.440000E 08   0.440000E 08   0.              0.              0.220000E 08   0.400000E-01
 2.      0.200000E 07   0.200000E 07   0.400000E 00    0.400000E 00    0.717900E 06   0.300000E 00

RING PROPERTIES                                        STRINGER PROPERTIES
 ER= 0.4400E 08      IR= 0.7812E-04                     ES=-0.              IS=-0.
 AR= 0.1500E-01    GRJR= C.3960E 03                     AS=-0.            GSJS=-0.
 ZR=-0.1250E 00       A= 0.3000E 01                     ZS=-0.               B=-0.

BASIC GEOMETRY
  L= 0.1200E 02    R= 0.602000E 01    DELTA= 0.20000E-01

MINIMUM P FOR N=0 IS  0.178153E 06 AT M=  5.

         M          RELATIVE MINIMA OF P          N
         1.            0.312903E 04               4.
         2.            0.625714E 04               5.
         3.            0.980885E 04               6.
         4.            0.135520E 05               6.
         5.            0.179774E 05               6.
         6.            0.230967E 05               6.
         7.            0.289914E 05               6.
         8.            0.357352E 05               6.
         9.            0.433762E 05               6.
        10.            0.519408E 05               6.

ABSOLUTE MINIMUM P=  0.312903E 04     M=  1.     N=  4.
```

Figure B-2. Example Computer Output

APPENDIX C

FORTRAN LISTING OF COMPUTER PROGRAM

```
C  ELASTIC BUCKLING OF SIMPLY SUPPORTED, ECCENTRICALLY STIFFENED CIRCULAR
C  CYLINDRICAL SHELLS WITH MULTIPLE ORTHOTROPIC LAYERS UNDER AXIAL COMPRESSION,
C  LATERAL PRESSURE OR HYDROSTATIC PRESSURE
C
C
C  READ STATEMENT FORMATS - -                                           BOLS
   1 FORMAT(80H                                                         BOLS
    1                                                                   BOLS
   2 FORMAT(I10,7F10.0)                                                 BOLS
   3 FORMAT(8E10.3)                                                     BOLS
C  WRITE STATEMENT FORMATS - -                                          BOLS
   4 FORMAT(90H ELASTIC BUCKLING OF SIMPLY SUPPORTED, ECCENTRICALLY STIBOLS
    1FFENED CIRCULAR CYLINDRICAL SHELLS/57H WITH MULTIPLE ORTHOTROPIC LBOLS
    2AYERS UNDER AXIAL COMPRESSION)                                     BOLS
   5 FORMAT(90H ELASTIC BUCKLING OF SIMPLY SUPPORTED, ECCENTRICALLY STIBOLS
    1FFENED CIRCULAR CYLINDRICAL SHELLS/56H WITH MULTIPLE ORTHOTROPIC LBOLS
    2AYERS UNDER LATERAL PRESSURE)                                      BOLS
   6 FORMAT(90H ELASTIC BUCKLING OF SIMPLY SUPPORTED, ECCENTRICALLY STIBOLS
    1FFENED CIRCULAR CYLINDRICAL SHELLS/60H WITH MULTIPLE ORTHOTROPIC LBOLS
    2AYERS UNDER HYDROSTATIC PRESSURE)                                  BOLS
   7 FORMAT(/4H MC=F4.0,5X3HMF=F4.0,5X3HNO=F4.0,5X3HNF=F4.0)            BOLS
   8 FORMAT(/15H PROPERTIES OF ,I1,19H ORTHOTROPIC LAYERS/6H LAYER,7X3HBOLS
    1EXX,12X3HEYY,12X4HNUXY,11X4HNUYX,11X3HGXY,12X1HT)                  BOLS
   9 FORMAT(F4.0,4XE13.6,5(2XE13.6))                                    BOLS
  10 FORMAT(/39H ORTHOTROPIC STIFFNESS LAYER PROPERTIES/5H  BX=E11.4,3XBOLS
    13HBY=E11.4,3X4HBXY=E11.4,3X6HNUXYB=E11.4/5H  DX=E11.4,3X3HDY=E11.4BOLS
    2,3X4HDXY=E11.4,3X6HNUXYD=E11.4,3X5HTOSL=E11.4)                     BOLS
  11 FORMAT(/16H RING PROPERTIES,32X19HSTRINGER PROPERTIES/5H  ER=E11.4BOLS
    1,5X3HIR=E11.4,15X3HES=E11.4,5X3HIS=E11.4/5H  AR=E11.4,3X5HGRJR=E11BOLS
    2.4,15X3HAS=E11.4,3X5HGSJS=E11.4/5H  ZR=E11.4,6X2HA=E11.4,15X3HZS=EBOLS
    311.4,6X2HB=E11.4)                                                  BOLS
  12 FORMAT(/15H BASIC GEOMETRY/5H   L=E11.4,3X2HR=E13.6,3X6HDELTA=E12.BOLS
    15)                                                                 BOLS
  13 FORMAT(/22H MINIMUM NX FOR N=0 IS,E14.6,6H AT M=F4.0/9X1HM,7X21HREBOLS
```

```
     1LATIVE MINIMA OF NX,7X1HM)
   14 FORMAT(/21H MINIMUM P FOR N=0 IS,E14.6,6H AT M=F4.0//9X1HM,7X20HRE
     1LATIVE MINIMA OF P,8X1HM)
   15 FORMAT(/9X1HM,7X20HRELATIVE MINIMA OF P,8X1HM)
   16 FORMAT(7XF4.0,8XE14.6,10XF4.0)
   17 FORMAT(/21H ABSOLUTE MINIMUM NX=E14.6,5X2HM=F4.0,5X2HN=F4.0)
   18 FORMAT(/20H ABSOLUTE MINIMUM P=E14.6,5X2HM=F4.0,5X2HN=F4.0)
C  ERROR MESSAGE FORMATS
   19 FORMAT(109H THE RELATIVE MINIMA ARE STILL DECREASING, SO THE RANGE
     1 ON M IS INSUFFICIENT TO DETERMINE AN ABSOLUTE MINIMUM/18H THE LAS
     2T VALUE IS,E14.6,6H AT M=F4.0)
   20 FORMAT(82H THE LOAD IS DECREASING, SO THE RANGE ON N IS INSUFFICIE
     1NT TO DETERMINE ALL MINIMA)
   21 FORMAT(/30H EQUAL OR NEAR EQUAL ORDINATES/7XF4.0,8H ORDNM1=E14.6,
     110XF4.0/7XF4.0,8H   ORDN=E14.6,10XF4.0//)
      DIMENSION K11(9),K12(9),K22(9),K33(9),DEL(9),EXX(9),EYY(9),NUXY(9)
     1,NUYX(9),GXY(9),T(9),LN(9)
      REAL IR,IS,M,MO,MF,MPL,N,NO,NF,NR,NUXY,NUYX,NUXYB,NUXYD,L,LOAD,
     1K11,K12,K22,K33,LN,NM1
      PI=3.14159265
C  READ INPUT DATA
  100 READ(5,1)
      READ(5,2)NL,OSL,LOAD,MO,MF,NO,NF
C  WRITE TITLE OF DATA AND PROBLEM
      WRITE(6,1)
C  WRITE TYPE OF LOADING AND RANGE ON M AND N
      IF(LOAD.EQ.1.) WRITE(6,4)
      IF(LOAD.EQ.2.) WRITE(6,5)
      IF(LOAD.EQ.3.) WRITE(6,6)
      WRITE(6,7) MO,MF,NO,NF
C  READ ORTHOTROPIC LAYER PROPERTIES
      DO 110 I=1,NL
  110 READ(5,3) LN(I),EXX(I),EYY(I), NUXY(I),NUYX(I),GXY(I),T(I)
      IF(OSL.EQ.1..AND.NL.EQ.1) GO TO 130
C  WRITE ORTHOTROPIC LAYER PROPERTIES
      WRITE(6,8) NL
      DO 120 I=1,NL
  120 WRITE(6,9) LN(I),EXX(I),EYY(I),NUXY(I),NUYX(I),GXY(I),T(I)
C  TEST FOR PRESENCE OF ORTHOTROPIC STIFFNESS LAYER
      IF(OSL.EQ.1.) GO TO 130
C  ZERO OUT PREVIOUS ORTHOTROPIC STIFFNESS LAYER PROPERTIES
      BX=0.
      BY=0.
      BXY=0.
      NUXYB=0.
      TOSL=0.
      DX=0.
      DY=0.
      DXY=0.
      NUXYD=0.
      GO TO 140
C  READ ORTHOTROPIC STIFFNESS LAYER PROPERTIES
  130 READ(5,3) BX,BY,BXY,NUXYB,TOSL
      T(1)=TOSL
      READ(5,3) DX,DY,DXY,NUXYD
C  WRITE ORTHOTROPIC STIFFNESS LAYER PROPERTIES
      WRITE(6,10) BX,BY,BXY,NUXYB,DX,DY,DXY,NUXYD,TOSL
C  READ AND WRITE RING AND STRINGER PROPERTIES
  140 READ(5,3) ER,AR,ZR,IR,GRJR,A
      READ(5,3) ES,AS,ZS,IS,GSJS,B
      WRITE(6,11) ER,IR,ES,IS,AR,GRJR,AS,GSJS,ZR,A,ZS,B
C  READ AND WRITE BASIC GEOMETRY
      READ(5,3) L,R,DELTA
      WRITE(6,12) L,R,DELTA
C  CALCULATE FUNCTIONS OF THE ELASTICITY CONSTANTS
      DO 150 I=1,NL
      K11(I)=EXX(I)/(1.-NUXY(I)*NUYX(I))
      K12(I)=NUXY(I)*K11(I)
```

```
      K22(I)=EYY(I)/(1.-NUXY(I)*NUYX(I))
  150 K33(I)=GXY(I)
C  CALCULATE DEL'S OF THE VARIOUS LAYERS
      DO 160 I=1,NL
      IF(I.EQ.1) DEL(I)=T(I)
      IF(I.NE.1) DEL(I)=DEL(I-1)+T(I)
  160 CONTINUE
C  ADJUST ZR AND ZS TO REFERENCE SURFACE
      IF(ZR.GT.0.) ZR = ZR+(DEL(NL)-DELTA)
      IF(ZS.GT.0.) ZS = ZS+(DEL(NL)-DELTA)
      IF(ZR.LT.0.) ZR = ZR-DELTA
      IF(ZS.LT.0.) ZS = ZS-DELTA
C  CALCULATE EXTENSIONAL, COUPLING, AND BENDING STIFFNESSES
C  ZERO OUT B'S, C'S, AND D'S PRIOR TO SUMMATION
      B11=0.
      B12=0.
      B22=0.
      B33=0.
      C11=0.
      C12=0.
      C22=0.
      C33=0.
      D11=0.
      D12=0.
      D22=0.
      D33=0.
      DO 190 I = 1,NL
      IF(I.NE.1) GO TO 170
      EXT=DEL(1)
      COUP=(1./2.)*(DEL(1)**2-2.*DELTA*DEL(1))
      BEND=(1./3.)*(DEL(1)**3-3.*DELTA*DEL(1)**2+3.*DELTA**2*DEL(1))
      GO TO 180
  170 EXT=DEL(I)-DEL(I-1)
      COUP=(1./2.)*((DEL(I)**2-DEL(I-1)**2)-2.*DELTA*(DEL(I)-DEL(I-1)))
      BEND=(1./3.)*((DEL(I)**3-DEL(I-1)**3)-3.*DELTA*(DEL(I)**2-DEL(I-1)
     1**2)+3.*DELTA**2*(DEL(I)-DEL(I-1)))
  180 B11=B11+K11(I)*EXT
      B12=B12+K12(I)*EXT
      B22=B22+K22(I)*EXT
      B33=B33+K33(I)*EXT
      C11=C11+K11(I)*COUP
      C12=C12+K12(I)*COUP
      C22=C22+K22 I)*COUP
      C33=C33+K33 I)*COUP
      D11=D11+K11 I)*BEND
      D12=D12+K12(I)*BEND
      D22=D22+K22(I)*BEND
  190 D33=D33+K33(I)*BEND
C  INITIALIZE
      ABSMIN=.7E35
      IF(LOAD.EQ.2.) GO TO 300
C  CALCULATE AXISYMMETRIC BUCKLING LOADS UNDER AXIAL OR HYDROS  TIC
C  LOADING FOR A RANGE OF MO TO 4*MF, AND PRINT MINIMUM LOAD
C  INITIALIZE
      AXIM=4.*MF
      M=MO
      ORDMM1=.8E35
      ORDMM2=.9E35
  200 MPL=M*PI/L
C  CALCULATE A VALUES
      A11=(B11+BX+ES*AS/B)*MPL**2
      A12=0.
      A13=(B12+NUXYB*BX)*MPL/R+(C11+ES*AS*ZS/B)*MPL**3
      A22=(B33+BXY)*MPL**2
      A23=0.
      A33=(D11+DX+ES*IS/B+ES*AS*ZS**2/B)*MPL**4+(2./R)*C12*MPL**2+(1./R*
     1*2)*(B22+BY+ER*AR/A)
      PART=A33+((A12*A23-A13*A22)/(A11*A22-A12**2))*A13+((A12*A1 )-A11*A2
```

```
     13)/(A11*A22-A12**2))*A23
C  TEST FOR TYPE OF LOADING, CALCULATE BUCKLING LOAD (NX OR PRESSURE),
C  AND STORE LOAD IN ADDRESS ORDM (ORDINATE AT ABSCISSA M)
      IF(LOAD.EQ.1.) ORDM=PART/MPL**2
      IF(LOAD.EQ.3.) ORDM=PART/(.5*MPL**2)
C  TEST FOR ABSOLUTE MINIMUM AXISYMMETRIC BUCKLING LOAD
C  ORDMM1 IS THE ORDINATE AT ABSCISSA M-1
C  ORDMM2 IS THE ORDINATE AT ABSCISSA M-2
C  TEST TO SEE WHETHER ORDM IS INCREASING OR DECREASING
      IF(ORDM.GT.ORDMM1) GO TO 210
C  ORDM DECREASING FROM OR EQUAL TO ORDMM1
      IF(M.EQ.AXIM) WRITE(6,19) ORDM,M
      GO TO 230
C  ORDM INCREASING FROM ORDMM1
  210 IF(ORDMM2.GT.ORDMM1) GO TO 220
C  NO RELATIVE MINIMUM FOUND
      GO TO 230
C  TEST FOR ABSOLUTE MINIMUM
  220 IF(ORDMM1.GT.ABSMIN) GO TO 230
C  NEW ABSOLUTE MINIMUM FOUND
      ABSMIN=ORDMM1
      ABSM=M-1.
      ABSN=0.
  230 IF(M.EQ.AXIM) GO TO 240
C  STEP M
      M=M+1.
      ORDMM2=ORDMM1
      ORDMM1=ORDM
      GO TO 200
C  WRITE AXISYMMETRIC BUCKLING LOAD
  240 IF(LOAD.EQ.1.) WRITE(6,13) ABSMIN,ABSM
      IF(LOAD.EQ.3.) WRITE(6,14) ABSMIN,ABSM
C  CALCULATE ASYMMETRIC BUCKLING LOADS FOR A SPECIFIED RANGE OF M AND N
C  INITIALIZE
  300 M=MO
      AMONM1=.5E35
      IF(LOAD.EQ.2.) WRITE(6,15)
C  BEGIN M LOOP
  310 MPL=M*PI/L
C  INITIALIZE FOR N LOOP
      N=NO
      ORDNM1=.8E35
      ORDNM2=.9E35
C  BEGIN N LOOP
  320 NR=N/R
C  CALCULATE A VALUES
      A11=(B11+BX+ES*AS/B)*MPL**2+(B33+BXY)*NR**2
      A12=(B12+NUXYB*BX+B33+BXY)*MPL*NR
      A13=(B12+NUXYB*BX)*MPL/R+(C11+ES*AS*ZS/B)*MPL**3+(C12+2.*C33)*MPL*
     1NR**2
      A22=(B33+BXY)*MPL**2+(B22+BY+ER*AR/A)*NR**2
      A23=(C12+2.*C33)*MPL**2*NR+(B22+BY+ER*AR/A)*NR/R+(C22+ER*AR*ZR/A)*
     1NR**3
      A33=(D11+DX+ES*IS/B+ES*AS*ZS**2/B)*MPL**4+(2.*(D12+NUXYD*DX)+4.*(D
     133+DXY)+GSJS/B+GRJR/A)*MPL**2*NR**2+(D22+DY+ER*IR/A+ER*AR*ZR**2/A)
     2*NR**4+(2./R)*C12*MPL**2+(2./R)*(C22+ER*AR*ZR/A)*NR**2+(1./R**2)*(
     3B22+BY+ER*AR/A)
      PART=A33+((A12*A23-A13*A22)/(A11*A22-A12**2))*A13+((A12*A13-A11*A2
     13)/(A11*A22-A12**2))*A23
C  TEST FOR TYPE OF LOADING, CALCULATE BUCKLING LOAD (NX OR PRESSURE),
C  AND STORE LOAD IN ADDRESS ORDN (ORDINATE AT ABSCISSA N)
      IF(LOAD.EQ.1.)ORDN=PART/MPL**2
      IF(LOAD.EQ.2.)ORDN=PART  R*NR**2)
      IF(LOAD.EQ.3.)ORDN=PART/(R*(.5*MPL**2+NR**2))
C  BEGIN TEST FOR RELATIVE MINIMA AND ABSOLUTE MINIMUM
C  ORDNM1 IS THE ORDINATE AT ABSCISSA N-1
C  ORDNM2 IS THE ORDINATE AT ABSCISSA N-2
C  TEST FOR EQUAL OR NEAR EQUAL ORDINATES
```

```
      IF(ABS(2.*(ORDN-ORDNM1)/(ORDN+ORDNM1)).GT..1E-3) GO TO 330
C  ORDINATES ARE CLOSE ENOUGH TO CAUSE TROUBLE IN THE SEARCH FOR
C  RELATIVE MINIMA, SO BEST INFORMATION IS TO WRITE ORDINATES
      NM1=N-1.
      WRITE(6,21) M,ORDNM1,NM1,M,ORDN,N
      GO TO 380
C  TEST TO SEE WHETHER ORDN IS INCREASING OR DECREASING
  330 IF(ORDN.GT.ORDNM1) GO TO 340
C  ORDN DECREASING
      IF(N.EQ.NF) WRITE(6,20)
      GO TO 380
C  ORDN INCREASING
  340 IF(ORDNM2.GT.ORDNM1) GO TO 350
C  NO RELATIVE MINIMUM
      GO TO 380
C  TEST FOR ABSOLUTE MINIMUM
C  AMONI IS THE ABSOLUTE MINIMUM VALUE OF ORDN IN THE MF-1 LOOP
  350 IF(M.EQ.MF-1..AND.ORDNM1.LT.AMONM1) AMONM1=ORDNM1
      IF(M.EQ.MF.AND.MO.NE.MF.AND.ORDNM1.LT.AMONM1) WRITE(6,19)
  360 IF(ORDNM1.GT.ABSMIN) GO TO 370
C  NEW ABSOLUTE MINIMUM FOUND
      ABSMIN=ORDNM1
      ABSM=M
      ABSN=N-1.
  370 RELMIN=ORDNM1
      RELN=N-1.
C  WRITE RELATIVE MINIMUM WITH CORRESPONDING M AND N
      WRITE(6,16)M,RELMIN,RELN
  380 IF(N.EQ.NF) GO TO 390
C  STEP N
      N=N+1.
      ORDNM2=ORDNM1
      ORDNM1=ORDN
      GO TO 320
  390 IF(M.EQ.MF) GO TO 395
C  STEP M
      M=M+1.
      GO TO 310
C  WRITE ABSOLUTE MINIMUM WITH CORRESPONDING M AND N
  395 IF(LOAD.EQ.1.) WRITE(6,17)ABSMIN,ABSM,ABSN
      IF(LOAD.EQ.2.) WRITE(6,18)ABSMIN,ABSM,ABSN
      IF(LOAD.EQ.3.) WRITE(6,18)ABSMIN,ABSM,ABSN
C  RETURN TO BEGINNING TO READ NEXT DATA CASE
      GO TO 100
      END
```

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APPENDIX D

BONDLESS, LAYERED SHELLS

The objective is to define a mathematical model for a circular cylindrical shell of multiple isotropic layers with no bond between the layers. This configuration is of interest as a lower bound to layered shells with shear-deformable bonds between the layers. The Kirchhoff-Love hypothesis is employed in all previous sections, but is valid only if the bonds between layers are non-shear-deformable. Accordingly, certain new definitions must be established. It is convenient to work within the framework of the orthotropic stiffness layer feature of the computer program (see Section A.2 of Appendix A). Certain stiffnesses and so-called Poisson's ratios must be defined, namely, quantities associated with extension (B_x, B_y, B_{xy}, and ν_{xyB}) and those associated with bending (D_x, D_y, D_{xy}, and ν_{xyD}).

The extensional stiffness of a layered shell is not affected by the presence or absence of a bond between the layers, i.e., it remains

$$B_x = B_y = \sum_{k=1}^{N} B_k \tag{D-1}$$

Similarly, the resistance to in-plane shear is unaffected, so

$$B_{xy} = \sum_{k=1}^{N} B_k (1 - \nu_k) /2 \tag{D-2}$$

If the force-strain relations are written in the form

$$\left.\begin{aligned} N_x &= \sum_{k=1}^{N} B_k\,(\epsilon_{xk} + \nu_k \epsilon_{yk}) \\ N_y &= \sum_{k=1}^{N} B_k\,(\epsilon_{yk} + \nu_k\, \epsilon_{xk}) \end{aligned}\right\} \tag{D-3}$$

and it is stipulated that the layers have the same strains, i.e.,

$$\left.\begin{aligned} \epsilon_{xk} &= \epsilon_x \\ \epsilon_{yk} &= \epsilon_y \end{aligned}\right\} \quad k = 1,\ N \tag{D-4}$$

then the so-called Poisson's ratio for extension can be identified as

$$\nu_{xyB} = \nu_B = \frac{\sum_{k=1}^{N} B_k\,\nu_k}{\sum_{k=1}^{N} B_k} \tag{D-5}$$

Note that ν_B is a geometrical as well as a material property.

The bending stiffness of a bondless, layered shell is the sum of the bending stiffnesses of the individual layers since the layers act with some measure of independence except for the requirement that the layers do not separate, i.e.,

$$D_x = D_y = \sum_{k=1}^{N} D_k \tag{D-6}$$

where D_k is the bending stiffness of the k^{th} layer about its own middle surface. Note that there are no terms such as occur in the transfer axis theorem for moments of inertia, i.e., no (area) times (distance squared) terms. Consequently, the bending stiffness is greatly decreased from the perfect bond case.

The consistent definition for the twisting stiffness follows from the stipulation that each layer independently resists twisting. Thus,

$$D_{xy} = \sum_{k=1}^{N} D_k (1 - \nu_k) / 2 \tag{D-7}$$

In analogy to the situation for extension, it is stipulated that the layers have the same changes in curvature, i.e.,

$$\left.\begin{aligned} \chi_{xk} &= \chi_x \\ \chi_{yk} &= \chi_y \end{aligned}\right\} \quad k = 1, N \tag{D-8}$$

Then the so-called Poisson's ratio for bending is obtained by use of the moment-change in curvature relations as

$$\nu_{xyD} = \nu_D = \frac{\sum_{k=1}^{N} D_k \nu_k}{\sum_{k=1}^{N} D_k} \tag{D-9}$$

Again, as with ν_B, ν_D is a geometrical as well as a material property.

The above approach implies that the layers have the same displacements and the same curvatures, i.e., all layers take the same shape. This implication is reasonable as long as the layers do not separate.

When the layers are in contact, the membrane circumferential strain is essentially the same in all layers if the sum of the layer thicknesses divided by the radius of the shell reference surface is small, i.e., a thin, layered shell. Thus, under lateral pressure, which is carried as membrane circumferential stress, σ_y, in the present buckling theory, σ_y in the k^{th} layer is proportional to the extensional stiffness of the k^{th} layer. Accordingly, the lateral pressure on each layer is given by

$$p_k = \frac{B_k}{\sum_{k=1}^{N} B_k} \cdot p \qquad \text{(D-10)}$$

where p is the lateral pressure on the layered shell. Thus, as a crude lower bound to the case of a bondless, layered shell, each layer must be thick enough to resist buckling under the pressure determined by Eq. (D-10). In addition, the layered shell with stiffnesses given by Eqs. (D-1), (D-2), (D-5), (D-6), (D-7), (D-9) must be thick enough to resist buckling under p.

Eccentrically stiffened, bondless, layered shells can be treated by appending stiffeners to the orthotropic stiffness layer in the manner discussed at the end of Section A.2 in Appendix A.

APPENDIX E

TWO-LAYERED, BONDLESS SHELLS WITH CIRCUMFERENTIAL CRACKS IN THE OUTER LAYER

The objective is to define a mathematical model for a circular cylindrical shell which has two unbonded, orthotropic layers and circumferential cracks in the outer layer (see Figure E-1). The principal axes of orthotropy must coincide with the shell coordinate axes. The orthotropic stiffness layer feature of the computer program (see Section A. 2 of Appendix A) is used in the calculations. Accordingly, certain stiffnesses and so-called Poisson's ratios must be defined, namely, quantities associated with extension (B_x, B_y, B_{xy}, and ν_{xyB}) and those associated with bending (D_x, D_y, D_{xy}, and ν_{xyD}).

Because of the circumferential cracks in the outer layer, the extensional stiffness in the axial direction is merely that of the inner layer, i. e., $B_{x2} = 0$. However, both layers are effective in resisting circumferential extension. Thus,

$$
\begin{aligned}
B_x &= B_{x1} \\
B_y &= B_{y1} + B_{y2}
\end{aligned}
\tag{E-1}
$$

No axial strain develops in the outer layer, i. e., $\epsilon_{x2} = 0$. Thus, the force-strain relations are

$$
\begin{aligned}
N_x &= B_{x1} (\epsilon_{x1} + \nu_{xyB1} \epsilon_{y1}) \\
N_y &= B_{y1} (\epsilon_{y1} + \nu_{yxB1} \epsilon_{x1}) + B_{y2} \epsilon_{y2}
\end{aligned}
\tag{E-2}
$$

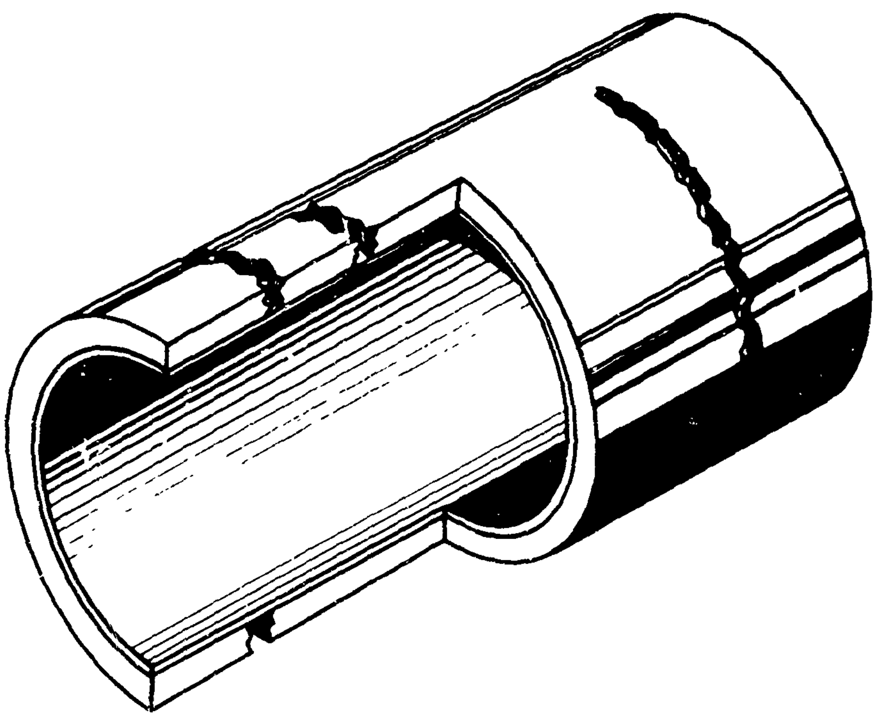

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Figure E-1. Cutaway View of a Two-Layered Circular Cylindrical Shell with (Exaggerated) Circumferential Cracks in the Outer Layer

Moreover, because the layers do not separate circumferentially,

$$\epsilon_{y1} \cong \epsilon_{y2} = \epsilon_y \tag{E-3}$$

Accordingly, the force-strain relations become

$$N_x = B_x (\epsilon_x + \nu_{xyB} \epsilon_y)$$

$$N_y = B_y (\epsilon_y + \nu_{yxB} \epsilon_x) \tag{E-4}$$

where B_x and B_y are defined in Eq. (E-1), and

$$\nu_{xyB} = \nu_{xyB1}$$

$$\nu_{yxB} = \nu_{yxB1} B_{y1} / (B_{y1} + B_{y2}) \tag{E-5}$$

Note that the reciprocal relations

$$\nu_{xyB} B_x = \nu_{yxB} B_y \tag{E-6}$$

are satisfied for the two-layered shell because they are satisfied for the inner layer, i. e.,

$$\nu_{xyB1} B_{x1} = \nu_{yxB1} B_{y1} \tag{E-7}$$

For an isotropic inner layer, Eq. (E-7) is an identity.

The inner layer carries all the in-plane shear because the outer layer is cracked. Thus,

$$B_{xy} = B_{xy1} \tag{E-8}$$

For an isotropic inner layer,

$$B_{xy1} = E_1 t_1 / 2(1 + \nu_1) \tag{E-9}$$

Reasoning parallel to the above leads to the following definitions for the quantities associated with bending.

$$\left.\begin{aligned} D_x &= D_{x1} \\ D_y &= D_{y1} + D_{y2} \end{aligned}\right\} \qquad (E\text{-}10)$$

$$\left.\begin{aligned} \nu_{xyD} &= \nu_{xyD1} \\ \nu_{yxD} &= \nu_{yxD1} \; D_{y1}/(D_{y1} + D_{y2}) \end{aligned}\right\} \qquad (E\text{-}11)$$

$$D_{xy} = D_{xy1} \qquad (E\text{-}12)$$

where, for an isotropic inner layer,

$$D_{xy1} = E_1 t_1^{\,3}/24\,(1 + \nu_1) \qquad (E\text{-}13)$$

In the definitions in Eqs. (E-10) to (E-12), it is implicit that

$$\chi_{y1} \cong \chi_{y2} = \chi_y \qquad (E\text{-}14)$$

in analogy to Eq. (E-3). Both Eqs. (E-3) and (E-14) are a result of no circumferential separation of layers. In addition, it should be noted that the bending stiffnesses of the layers in Eq. (E-16) are about the middle surface of the respective layers because of the lack of bonding between layers.

Eccentrically stiffened, bondless, layered shells with circumferential cracks can be treated by appending stiffeners to the orthotropic stiffness layer in the manner discussed at the end of Section A. 2 of Appendix A.

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13 ABSTRACT

An exact solution is derived for the buckling of a circular cylindrical shell with multiple orthotropic layers and eccentric stiffeners under axial compression, lateral pressure, or any combination thereof. Classical stability theory (membrane prebuckled shape) is used for simply supported edge boundary conditions. The present theory enables the study of coupling between bending and extension due to the presence of different layers in the shell and to the presence of eccentric stiffeners. Previous approaches to stiffened multilayered shells are shown to be erratic in the prediction of buckling results due to neglect of coupling between bending and extension. (Unclassified Report)

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Aerospace Report No. TR-0158(S3820-10)-1,
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1. Delete Eq. (D-10) and the discussion in the surrounding paragraph on page 38 as the shell buckling analysis is unduly conservative if the deleted considerations are imposed. That is, an inner layer when constrained by an outer layer would be expected to buckle at a very much higher load than that of the unrestrained shell implied by the deleted considerations. The buckling load of the constrained shell would be expected to be higher than that determined by the model discussed in Appendix D. Thus, the model in Appendix D appears to be the most reasonable model which could be devised.

2. Replace Appendix E (pages 39 through 42) with the attached revised pages.

T. A. Bergstralh, General Manager
Technology Division

APPENDIX E

TWO-LAYERED, BONDLESS SHELLS WITH CIRCUMFERENTIAL CRACKS IN THE OUTER LAYER

The objective is to define a mathematical model for a circular cylindrical shell which has two unbonded, orthotropic layers and circumferential cracks in the outer layer (see Figure E-1). The principal axes of orthotropy must coincide with the shell coordinate axes. The orthotropic stiffness layer feature of the computer program (see Section A.2 of Appendix A) is used in the calculations. Accordingly, certain stiffnesses and so-called Poisson's ratios must be defined, namely, quantities associated with extension (B_x, B_y, B_{xy}, and ν_{xyB}) and those associated with bending (D_x, D_y, D_{xy}, and ν_{xyD}).

Because of the circumferential cracks in the outer layer and the lack of bonds between layers, the axial force in the outer layer is zero, i.e.,

$$N_{x2} = B_{x2} (\epsilon_{x2} + \nu_{xyB2} \epsilon_{y2}) = 0 \tag{E-1}$$

The remaining segments of the outer layer are analogous to plane stress ring elements, the axial stiffness of which is finite. Accordingly, from Eq. (E-1),

$$\epsilon_{x2} = -\nu_{xyB2} \epsilon_{y2} \tag{E-2}$$

The force-strain relations can then be written as

$$\begin{aligned} N_x &= B_{x1} (\epsilon_{x1} + \nu_{xyB1} \epsilon_{y1}) \\ N_y &= B_{y1} (\epsilon_{y1} + \nu_{yxB1} \epsilon_{x1}) + B_{y2} (\epsilon_{y2} + \nu_{yxB2} \epsilon_{x2}) \end{aligned} \tag{E-3}$$

Moreover, because the layers do not separate circumferentially,

$$\epsilon_{y1} \cong \epsilon_{y2} = \epsilon_y \tag{E-4}$$

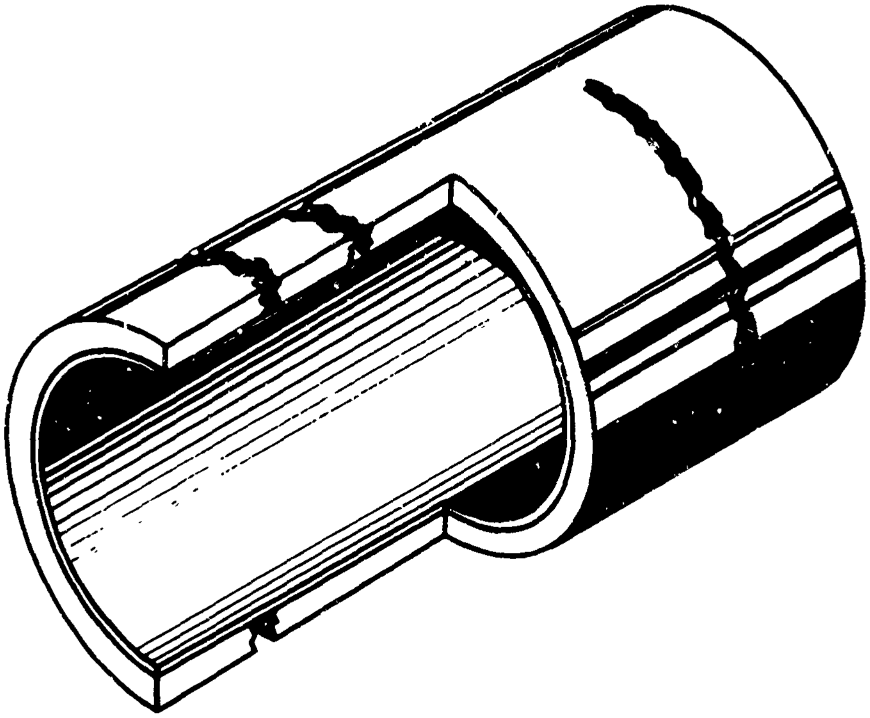

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Figure E-1. Cutaway View of a Two-Layered Circular Cylindrical Shell with (Exaggerated) Circumferential Cracks in the Outer Layer whereupon, with Eq. (E-2), the force strain relations become

$$N_x = B_x (\epsilon_x + \nu_{xyB} \epsilon_y)$$
$$N_y = B_y (\epsilon_y + \nu_{yxB} \epsilon_x) \qquad \text{(E-5)}$$

where

$$B_x = B_{x1}$$
$$B_y = B_{y1} + B_{y2} (1 - \nu_{yxB2} \nu_{xyB2}) \qquad \text{(E-6)}$$
$$\epsilon_x = \epsilon_{x1}$$

and

$$\nu_{xyB} = \nu_{xyB1}$$
$$\nu_{yxB} = \nu_{yxB1} B_{y1}/B_y \qquad \text{(E-7)}$$

Note that the reciprocal relations

$$\nu_{xyB} B_x = \nu_{yxB} B_y \qquad \text{(E-8)}$$

are satisfied for the two-layered shell because they are satisfied for the inner layer, i.e.,

$$\nu_{xyB1} B_{x1} = \nu_{yxB1} B_{y1} \qquad \text{(E-9)}$$

For an isotropic inner layer, Eq. (E-9) is an identity.

The inner layer carries all the in-plane shear because the outer layer is cracked. Thus,

$$B_{xy} = B_{xy1} \qquad \text{(E-10)}$$

For an isotropic inner layer,

$$B_{xy1} = E_1 t_1/2(1 + \nu_1) \qquad \text{(E-11)}$$

Reasoning parallel to the above leads to the following definitions for the quantities associated with bending:

$$D_x = D_{x1}$$

$$D_y = D_{y1} + D_{y2} (1 - \nu_{yxD2} \nu_{xyD2}) \qquad (E-12)$$

$$D_{xy} = D_{xy1}$$

and

$$\nu_{xyD} = \nu_{xyD1}$$

$$\nu_{yxD} = \nu_{yxD1} D_{y1}/D_y \qquad (E-13)$$

where, for an isotropic layer,

$$D_{xy1} = E_1 t_1^3 / 24(1 + \nu_1) \qquad (E-14)$$

In the definitions in Eqs. (E-12) and (E-13), it is implicit that

$$\chi_{y1} \cong \chi_{y2} = \chi_y \qquad (E-15)$$

and

$$\chi_{x1} = \chi_x \qquad (E-16)$$

in analogy to Eqs. (E-4) and (E-6). Both Eqs. (E-4) and (E-15) are a result of no circumferential separation of layers. In addition, it should be noted that the bending stiffnesses of the layers in Eq. (E-12) are about the middle surface of the respective layers because of the lack of bonding between layers.

Eccentrically stiffened, bondless, layered shells with circumferential cracks can be treated by appending stiffeners to the orthotropic stiffness layer in the manner discussed at the end of Section A. 2 of Appendix A.